

11.2 | 연습문제 정답

01.

$$(a) \frac{\partial f}{\partial x} = y - 1, \quad \frac{\partial f}{\partial y} = x$$

$$(b) \frac{\partial f}{\partial x} = 2(x+1)(4y-1)^3, \quad \frac{\partial f}{\partial y} = 12(x+1)^2(4y-1)^2$$

$$(c) \frac{\partial f}{\partial x} = \cos(x+y), \quad \frac{\partial f}{\partial y} = \cos(x+y)$$

$$(d) \frac{\partial f}{\partial x} = \frac{1}{1 + \left(\frac{1}{x^2 + y^2}\right)^2} \frac{-2x}{(x^2 + y^2)^2} = \frac{-2x}{(x^2 + y^2)^2 + 1}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1 + \left(\frac{1}{x^2 + y^2}\right)^2} \frac{-2y}{(x^2 + y^2)^2} = \frac{-2y}{(x^2 + y^2)^2 + 1}$$

$$(e) f(u, v) = e^{\frac{v}{u}} (\cos u + \sin u)$$

$$\frac{\partial f}{\partial u} = -\frac{v}{u^2} e^{\frac{v}{u}} (\cos u + \sin u) + e^{\frac{v}{u}} (-\sin u + \cos u)$$

$$\frac{\partial f}{\partial v} = \frac{1}{u} e^{\frac{v}{u}} (\cos u + \sin u)$$

$$(f) \frac{\partial f}{\partial r} = 1 - \cos \theta, \quad \frac{\partial f}{\partial \theta} = r \sin \theta$$

$$(g) \frac{\partial f}{\partial \phi} = \cos \phi \cos \theta, \quad \frac{\partial f}{\partial \theta} = -\sin \phi \sin \theta$$

$$(h) \frac{\partial f}{\partial x} = \frac{2xy - (2x - y)y}{(xy)^2} = \frac{1}{x^2}, \quad \frac{\partial f}{\partial y} = \frac{-xy - (2x - y)x}{(xy)^2} = -\frac{2}{y^2}$$

$$(i) \frac{\partial f}{\partial w} = \sin^{-1}\left(\frac{w}{z}\right) + \frac{w}{\sqrt{1 - \left(\frac{w}{z}\right)^2}} \left(\frac{1}{z}\right) = \sin^{-1}\left(\frac{w}{z}\right) + \frac{w}{z \sqrt{1 - \left(\frac{w}{z}\right)^2}}$$

$$\frac{\partial f}{\partial z} = \frac{w}{\sqrt{1 - \left(\frac{w}{z}\right)^2}} \left(-\frac{w}{z^2}\right) = -\frac{w^2}{z^2 \sqrt{1 - \left(\frac{w}{z}\right)^2}}$$

$$(j) \quad \frac{\partial f}{\partial s} = \frac{2s}{s^2 - t^2}, \quad \frac{\partial f}{\partial t} = \frac{-2t}{s^2 - t^2}$$

02.

$$(a) \quad \frac{\partial f}{\partial x} = 6x^2y^4 - 15x^4y^3 \Rightarrow \frac{\partial^2 f}{\partial y \partial x} = 24x^2y^3 - 45x^4y^2$$

$$\frac{\partial f}{\partial y} = 8x^3y^3 - 9x^5y^2 \Rightarrow \frac{\partial^2 f}{\partial x \partial y} = 24x^2y^3 - 45x^4y^2$$

$$(b) \quad \frac{\partial f}{\partial x} = \frac{2x}{x^2 + y^2} \Rightarrow \frac{\partial^2 f}{\partial y \partial x} = \frac{-4xy}{(x^2 + y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{2y}{x^2 + y^2} \Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{-4xy}{(x^2 + y^2)^2}$$

$$(c) \quad \frac{\partial f}{\partial x} = \frac{y}{1 + (xy)^2} \Rightarrow \frac{\partial^2 f}{\partial y \partial x} = \frac{(1 + x^2y^2) - y(2x^2y)}{(1 + x^2y^2)^2} = \frac{1 - x^2y^2}{(1 + x^2y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{x}{1 + (xy)^2} \Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{(1 + x^2y^2) - x(2xy^2)}{(1 + x^2y^2)^2} = \frac{1 - x^2y^2}{(1 + x^2y^2)^2}$$

$$(d) \quad \frac{\partial f}{\partial x} = 3e^{3x} \cos y \Rightarrow \frac{\partial^2 f}{\partial y \partial x} = -3e^{3x} \sin y$$

$$\frac{\partial f}{\partial y} = -e^{3x} \sin y \Rightarrow \frac{\partial^2 f}{\partial x \partial y} = -3e^{3x} \sin y$$

03. $\frac{64}{25}$

04. 10

05. $-\frac{1}{32}$

06.

$$(a) \quad \frac{\partial f}{\partial x} = 2x, \quad \frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial f}{\partial y} = 2y, \quad \frac{\partial^2 f}{\partial y^2} = 2, \quad \frac{\partial f}{\partial z} = -4z, \quad \frac{\partial^2 f}{\partial z^2} = -4$$

$$\text{이므로 } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0 \text{이다.}$$

$$(b) \quad \frac{\partial f}{\partial x} = \frac{\frac{-x}{\sqrt{x^2+y^2+z^2}}}{\left(\sqrt{x^2+y^2+z^2}\right)^2} = \frac{-x}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{\left(\sqrt{x^2+y^2+z^2}\right)^3 - 3x^2 \sqrt{x^2+y^2+z^2}}{\left(\sqrt{x^2+y^2+z^2}\right)^6} = \frac{2x^2 - y^2 - z^2}{\left(\sqrt{x^2+y^2+z^2}\right)^5}$$

$$\frac{\partial f}{\partial y} = \frac{\frac{-y}{\sqrt{x^2+y^2+z^2}}}{\left(\sqrt{x^2+y^2+z^2}\right)^2} = \frac{-y}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

$$\frac{\partial^2 f}{\partial y^2} = -\frac{\left(\sqrt{x^2+y^2+z^2}\right)^3 - 3y^2 \sqrt{x^2+y^2+z^2}}{\left(\sqrt{x^2+y^2+z^2}\right)^6} = \frac{-x^2 + 2y^2 - z^2}{\left(\sqrt{x^2+y^2+z^2}\right)^5}$$

$$\frac{\partial f}{\partial z} = \frac{\frac{-z}{\sqrt{x^2+y^2+z^2}}}{\left(\sqrt{x^2+y^2+z^2}\right)^2} = \frac{-z}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

$$\frac{\partial^2 f}{\partial z^2} = -\frac{\left(\sqrt{x^2+y^2+z^2}\right)^3 - 3z^2 \sqrt{x^2+y^2+z^2}}{\left(\sqrt{x^2+y^2+z^2}\right)^6} = \frac{-x^2 - y^2 + 2z^2}{\left(\sqrt{x^2+y^2+z^2}\right)^5}$$

$$\text{이므로 } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0 \text{이다.}$$

$$(c) \quad \frac{\partial f}{\partial x} = 3e^{3x+4y} \cos(5z), \quad \frac{\partial^2 f}{\partial x^2} = 9e^{3x+4y} \cos(5z)$$

$$\frac{\partial f}{\partial y} = 4e^{3x+4y} \cos(5z), \quad \frac{\partial^2 f}{\partial y^2} = 16e^{3x+4y} \cos(5z)$$

$$\frac{\partial f}{\partial z} = -5e^{3x+4y} \sin(5z), \quad \frac{\partial^2 f}{\partial z^2} = -25e^{3x+4y} \cos(5z)$$

$$\text{이므로 } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0 \text{이다.}$$

$$(d) \frac{\partial f}{\partial x} = 3x^2y - y^3, \quad \frac{\partial^2 f}{\partial x^2} = 6xy, \quad \frac{\partial f}{\partial y} = x^3 - 3xy^2, \quad \frac{\partial^2 f}{\partial y^2} = -6xy, \quad \frac{\partial f}{\partial z} = \frac{\partial^2 f}{\partial z^2} = 0$$

$$\text{이므로 } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0 \text{이다.}$$

$$(e) \frac{\partial f}{\partial x} = \frac{2x}{x^2 + y^2}, \quad \frac{\partial^2 f}{\partial x^2} = \frac{2\{(x^2 + y^2) - 2x^2\}}{(x^2 + y^2)^2} = \frac{-2x^2 + 2y^2}{(x^2 + y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{2y}{x^2 + y^2}, \quad \frac{\partial^2 f}{\partial y^2} = \frac{2\{(x^2 + y^2) - 2y^2\}}{(x^2 + y^2)^2} = \frac{2x^2 - 2y^2}{(x^2 + y^2)^2}, \quad \frac{\partial f}{\partial z} = \frac{\partial^2 f}{\partial z^2} = 0$$

$$\text{이므로 } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0 \text{이다.}$$

$$(f) \frac{\partial f}{\partial x} = -2e^{-2y} \sin(2x) + \frac{1}{1 + \left(\frac{y}{x}\right)^2} \left(\frac{-y}{x^2}\right) = -2e^{-2y} \sin(2x) - \frac{y}{x^2 + y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = -4e^{-2y} \cos(2x) + \frac{2xy}{(x^2 + y^2)^2}$$

$$\frac{\partial f}{\partial y} = -2e^{-2y} \cos(2x) + \frac{1}{1 + \left(\frac{y}{x}\right)^2} \left(\frac{1}{x}\right) = -2e^{-2y} \cos(2x) + \frac{x}{x^2 + y^2}$$

$$\frac{\partial^2 f}{\partial y^2} = 4e^{-2y} \cos(2x) - \frac{2xy}{(x^2 + y^2)^2}, \quad \frac{\partial f}{\partial z} = \frac{\partial^2 f}{\partial z^2} = 0$$

$$\text{이므로 } \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0 \text{이다.}$$

07. (1, 0, 29)

08. 수 $f(x, y) = x^3 + y^3$ 는 (1, 1)에서 미분가능이다.

09.

$$(a) f_x(x, y) = \begin{cases} \frac{4x^3(x^2 + y^2) - (x^4 + y^4)(2x)}{(x^2 + y^2)^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

$$f_y(x,y) = \begin{cases} \frac{4y^3(x^2+y^2) - (x^4+y^4)(2y)}{(x^2+y^2)^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(b) $f_x(x,y)$ 는 $(0,0)$ 에서 연속이다. $f_y(x,y)$ 는 $(0,0)$ 에서 연속이다.

(c) $f(x,y)$ 는 $(0,0)$ 에서 미분가능하다.

10. (a) $f_x(0,0) = 0$, $f_y(0,0) = 0$

(b) $f(x,y)$ 는 $(0,0)$ 에서 미분 불가능하다.

11. (a) $f_x(0,0) = 0$, $f_y(0,0) = 0$

(b) f 는 $(0,0)$ 에서 미분가능하다.

12. 7%